



## Coordinate transformation learning of hand position feedback controller with time delay

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### Abstract

The speed, accuracy, and adaptability of human movement depends on the brain performing an inverse kinematics transformation—i.e., a transformation from visual to joint angle coordinates—based on learning from experience. In human visually guided motion control, it is important to learn a feedback controller for the hand position error. This paper proposes two novel models that learn coordinate transformations of the human visual feedback controller with time delay. The proposed models redress drawbacks in current models because they do not rely on complex signal switching, which does not seem neurophysiologically plausible. © 2001 Published by Elsevier Science B.V.

*Keywords:* Coordinate transformation learning; Visual feedback; Time delay; Learning control; Inverse kinematics

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### 1. Introduction

An infant without a thumb had a major surgical operation, transplanting an index finger as a thumb, which is kinematically influential. After the operation, the child was able to learn how to use the index finger like a thumb [6]. We believe that the coordinate transformation learning of the visual feedback controller is necessary to explain the motor learning capability of humans. Although a number of learning

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models of the visual feedback controller have been proposed [3,1], a definitive learning model has not yet been obtained. Many researchers [1] employ *direct inverse modeling*. However, it requires the complex switching of the input signal to the inverse model from the *desired* hand position, velocity, or acceleration during hand position control in the visual coordinates to the *observed* hand position, velocity, or acceleration during controller learning. Although the desired and observed signal might coincide, their characteristics are quite different. No research has yet modeled the switching system successfully. Furthermore, the learning model is not “goal-directed” [3]. The *forward and inverse modeling* proposed by Jordan [3] requires a backpropagation signal; this technique lacks a biological basis. It also requires complex switching of the desired *output* signal for the forward model from the observed hand position in the visual coordinates during forward model learning to the desired hand position during controller learning. We believe that the complex signal switching for the learning required by *direct inverse modeling* or *forward and inverse modeling* does not occur in the relatively low-level sensorimotor learning of the human nervous system. The *feedback error learning* proposed by Kawato [5] requires a pre-existing accurate feedback controller. We have already proposed two models for learning the coordinate transformation function of the visual feedback controller. One is based on disturbance noise in the hand position control loop [8]. The other is based on changes in hand position error [9]. Both can avoid complex signal switching. However, they do not pay enough attention to the time delay in the visual feedback control loop. This paper presents improved learning models of the coordinate transformation function of the visual feedback controller with time delay [7].

## 2. Background

Let  $\theta \in \mathbb{R}^m$  be the joint angle/muscle length vector and  $x \in \mathbb{R}^n$  be the hand position/orientation vector in the visual coordinates given by the vision system. The relationship between  $x$  and  $\theta$  is expressed as  $x = f(\theta)$ . The Jacobian of the hand position vector is expressed as  $J(\theta) = \partial f(\theta) / \partial \theta$ . Let  $x_d(k)$  ( $k = -\gamma T, \dots, -1, 0, 1, 2, \dots, T$ ) be the desired hand position and  $e(k) = x_d(k) - x(k) = x_d(k) - f(\theta(k))$  be the hand position error vector.  $\gamma$  is an appropriate constant. Let  $\Phi_{ff}(\theta(k), \Delta x_d(k))$  be the output of the feed-forward controller that transforms  $\Delta x_d(k) = x_d(k+1) - x_d(k)$  to the joint angle/muscle length vector space and  $\Phi_{fb}(\theta(k), e(k-\tau))$  be the output of the error feedback controller. Because of the time delay in visual information processing, the feedback controller at time step  $k$  handles the hand position error at time step  $k - \tau$ . The following control system is considered:

$$\theta(k+1) = \theta(k) + \Delta\theta(k), \quad (1)$$

$$\Delta\theta(k) = d(k) + \Delta\theta_{ff}(k) + \Delta\theta_{fb}(k), \quad (2)$$

$$\Delta\theta_{ff}(k) = \Phi_{ff}(\theta(k), \Delta x_d(k)), \quad (3)$$

$$\Delta\theta_{fb}(k) = \Phi_{fb}(\theta(k), e(k-\tau)), \quad (4)$$

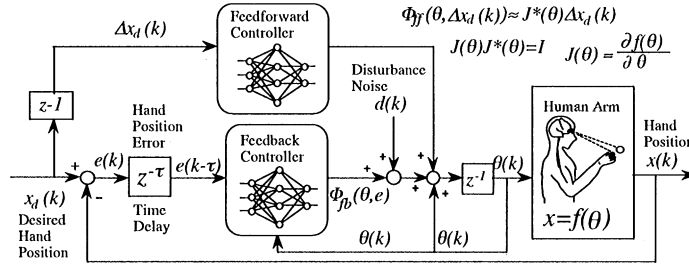


Fig. 1. Configuration of hybrid control system.

where  $d(k)$  is assumed to be a disturbance noise from all components except the visual feedback control system. Fig. 1 shows the configuration of the control system. The hand position error is updated as follows:

$$e(k + 1) \approx e(k) + \Delta x_d(k) - \Delta x(k),$$

$$\approx e(k) + e_{ff}(k) - J(\theta(k))d(k) + \Delta \theta_{fb}(k), \tag{5}$$

$$e_{ff}(k) = \Delta x_d(k) - J(\theta(k)) \Phi_{ff}(\theta(k), \Delta x_d(k)). \tag{6}$$

We call the following term the modified feedback error signal.

$$\Phi_{fb}^*(k) = \Delta \theta_{fb}(k + 1) - \Delta \theta_{fb}(k) + K(d(k - \tau) + \Delta \theta_{fb}(k - \tau)), \tag{7}$$

where  $K$  is an appropriate positive real number. Let  $\Psi_{fb}(\theta)$  be an appropriate coordinate transformation gain. If the feedback controller is learned as  $\Phi_{fb}(\theta, e) \approx \Psi_{fb}(\theta, e)e$ , the following approximate equation is obtained.

$$\Phi_{fb}^*(k) \approx \Psi_{fb}(\theta) e_{ff}(k - \tau) + \Psi_{fb}(\theta) e_{fb}(k - 2\tau)$$

$$+ (KI_m - \Psi_{fb}(\theta)J(\theta))d(k - \tau), \tag{8}$$

$$e_{fb}(k) = Ke(k) - J(\theta(k + \tau))\Phi_{fb}(\theta(k + \tau), e(k)). \tag{9}$$

Using Kawato's *Feedback error learning* [5], the first term of the right-hand side of Eq. (8) can reduce the error of the feed-forward controller  $e_{ff}$ . We use the desired output for the feed-forward controller expressed as

$$\Phi_{ff}'(\theta(k), \Delta x_d(k)) = (1 - \lambda_{ff})\Phi_{ff}(\theta(k), \Delta x_d(k)) + \Phi_{fb}^*(k + \tau), \tag{10}$$

where  $\lambda_{ff}$  is a small, positive, real number for stabilizing the learning process and ensuring that equation  $\Phi_{ff}(\theta, 0) \approx 0$  holds. If  $\lambda_{ff}$  is small enough, the learning feed-forward controller will fulfill  $J\Phi_{ff}'(\theta, \Delta x_d) \approx \Delta x_d$ .

### 3. Learning based on disturbance noise and feedback error signal

One role of the feedback controller is to compensate for disturbance noise. We proposed the learning model based on the disturbance noises in the hand position

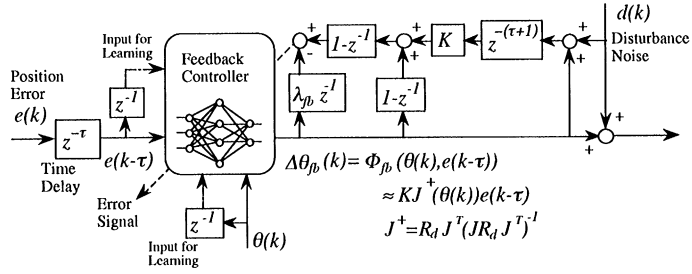


Fig. 2. Learning based on disturbance noise and feedback error signal.

control loop [8]. There are a variety of sources of the disturbance noises in human motion control system. Infants experience various kinds of motions including reflexes before they can reach and grasp objects [4]. Motion signals that are not generated by the visual feedback control system can be regarded as disturbance noise. Based on observations of motor-neural firing, Harris and Wolpert assumed that the neural control signal contains noise that increases with the mean of the signal [2]. A lack of completeness in the inverse dynamics computation in human motion control system can cause the desired and real joint motion to differ. The error can be regarded as disturbance noise. The disturbance noise and the feedback error signal are useful for learning the feedback controller [8].

Considering the time delay, we modified the learning model as follows:

$$\Phi'_{fb}(\theta(k), e(k - \tau)) = (1 - \lambda_{fb})\Phi_{fb}(\theta(k), e(k - \tau)) + \Phi_{fb}^*(k) - \Phi_{fb}^*(k - 1), \quad (11)$$

where  $\lambda_{fb}$  is a small, positive, real number. This learning is conducted at time step  $k + 1$ .  $\Phi_{fb}^*(k)$  in Eq. (11) contains  $\Psi_{fb}(\theta)e_{fb}(k - 2\tau)$  which is effective for the learning of  $\Phi_{fb}(\theta(k), e(k - \tau))$ .  $-\Phi_{fb}^*(k - 1)$  contains  $-Kd(k - \tau - 1)$  and makes  $\Phi_{fb}(\theta(k), e(k - \tau))$  to compensate  $d(k - \tau - 1)$ . Fig. 2 shows the conceptual diagram of the proposed learning model. We assume that  $d(k)$  is not  $\mathbf{0}$ . Based on some feasible assumptions, the learning result is expressed by

$$\Phi_{fb}(\theta, e) \approx KJ^+(\theta)e = KR_d J^T (\mathbf{J}(\theta) R_d J^T(\theta))^{-1} e, \quad (12)$$

where  $R_d$  is the covariance matrix of  $d(k)$  defined as  $R_d = E[dd^T]$ .  $E[\mathbf{t}]$  is the expectation value of a scalar, a vector, or a matrix function  $\mathbf{t}$ .  $J^+(\theta)$  is the pseudo-inverse matrix (Moore–Penrose' generalized inverse matrix) of  $\mathbf{J}(\theta)$ .

#### 4. Learning based on change of hand position error norm

We proposed the alternative learning model that uses the product of the change of joint angle/muscle length vector and the corresponding change of square of hand position error norm in the visual coordinates [9]. Fig. 3 shows the conceptual

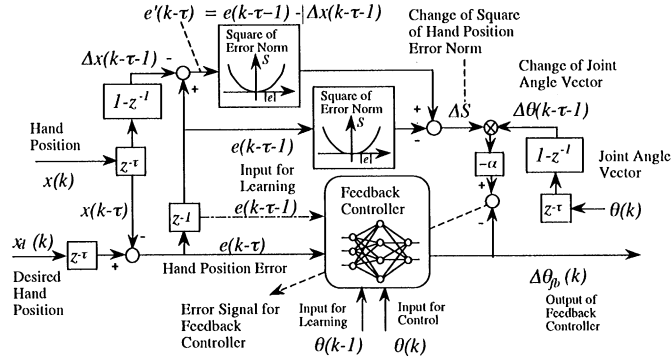


Fig. 3. Learning based on change of error.

diagram of the proposed learning model. Let  $S(x_d, \theta)$  be the square of the error norm defined as  $S(x_d, \theta) = |x_d - f(\theta)|^2/2$ . The change of the square of the hand position error norm  $\Delta S = S(x_d, \theta + \Delta\theta) - S(x_d, \theta)$  reflects whether or not the change of the joint angle vector  $\Delta\theta$  is in proper direction. When the input to the feedback controller is  $(\theta, e)$ , the expectation value of  $\Delta S \Delta\theta$  can be expressed as follows:

$$E[\Delta S \Delta\theta | \theta, e] \approx -R_\theta J^T e + \frac{1}{2} R_\theta J^T J \Phi_{fb}(\theta, e) + \frac{1}{2} E[\Delta\theta \Delta\theta^T J^T J \Delta\theta_{nfb} | \theta, e], \quad (13)$$

$$\Delta\theta_{nfb} = \Delta\theta - \Delta\theta_{fb}, \quad (14)$$

where  $R_\theta$  is the covariance matrix of  $\Delta\theta$  defined as  $R_\theta = E[\Delta\theta \Delta\theta^T | \theta, e]$ .  $E[\mathcal{f}(s) | s]$  is the expectation value of a scalar, a vector, or a matrix function  $\mathcal{f}(s)$  when the input to the function is  $s$ . The first term of the right-hand side of Eq. (13) can be used for the coordinate transformation. The proposed learning model can be expressed as follows:

$$\Phi'_{fb}(\theta(k), e(k - \tau)) = -\alpha \Delta S(k - \tau) \Delta\theta(k - \tau), \quad (15)$$

$$\Delta S(k) = \frac{1}{2} (|x_d(k) - x(k + 1)|^2 - |x_d(k) - x(k)|^2), \quad (16)$$

where  $\alpha$  is a small positive real number. This learning is conducted at time step  $k + 1$ . Based on some feasible assumptions, the result of the learning can be expressed as:

$$\begin{aligned} \Phi_{fb}(\theta, e) &\approx \alpha \left( \frac{\alpha}{2} ((R_\theta + R_{nfb}) J^T J + r_{Jnfb} I_m) + I_m \right)^{-1} R_\theta J^T e \\ &\approx \alpha \left( \frac{\alpha}{2} (2R_{nfb} J^T J + r_{J\Delta\theta} I_m) + I_m \right)^{-1} R_\theta J^T e \end{aligned} \quad (17)$$

$$R_{nfb} = E[\Delta\theta_{nfb} \Delta\theta_{nfb}^T | \theta, e] \quad (18)$$

$$r_{Jnfb} = E[|J(\theta) \Delta\theta_{nfb}|^2 | \theta, e] \quad (19)$$

$$r_{J\Delta\theta} = E[|J(\theta) \Delta\theta|^2 | \theta, e] \quad (20)$$

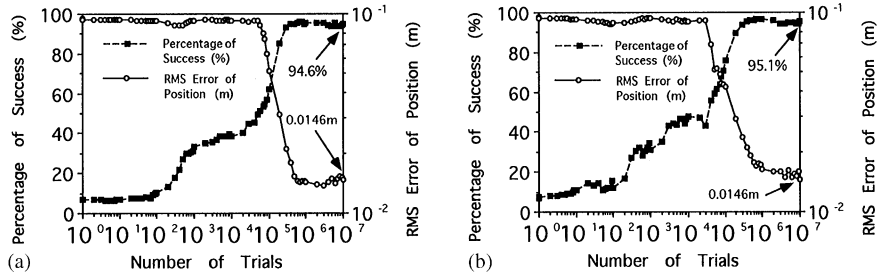


Fig. 4. Simulation results (a) Learning based on disturbance noise and feedback error signal. (b) Learning based on change of hand position error.

where  $\mathbf{J}^T(\boldsymbol{\theta})\mathbf{e}$  is a vector in the steepest descent direction of  $S(\mathbf{x}_d, \boldsymbol{\theta})$ . When  $\alpha$  is appropriate,  $\Phi_{fb}(\boldsymbol{\theta}, \mathbf{e})$  as expressed in Eq. (17) can provide appropriate output error feedback control.

## 5. Numerical experiments

We performed simulations of the coordinate transformation learning of the feedback controller of a human-like 7-DOF arm. The arm length was 0.755 m. Random straight lines provided the desired trajectories for the hand. The tracking control trials with online incremental learning were performed. The time delay  $\tau$  was 4,  $T$  was 40 and  $\gamma$  was 0,  $K$  was set at 0.2. When the hand position error grew larger than 0.1 m, the trial was regarded as a failure.

In order to accelerate the learning, neural networks that have a structure suitable for the coordinate transformation as  $\Phi_{fb}(\boldsymbol{\theta}, \mathbf{e}) = \Psi_{fb}(\boldsymbol{\theta})\mathbf{e}$  were used. The desired output signal for the coordinate transformation gain of the feedback controller  $\Psi'_{fb}(\boldsymbol{\theta})$  was expressed as follows:

$$\Psi'_{fb}(\boldsymbol{\theta}) = \Psi_{fb}(\boldsymbol{\theta}) + \frac{(\Phi'_{fb}(\boldsymbol{\theta}, \mathbf{e}) - \Phi_{fb}(\boldsymbol{\theta}, \mathbf{e}))\mathbf{e}^T}{0.01 + |\mathbf{e}|\Phi'_{fb}(\boldsymbol{\theta}, \mathbf{e})}, \quad (21)$$

$\Psi_{ff}(\boldsymbol{\theta})$  was also updated in the same manner described in the above equation.  $\alpha$  in Eq. (15) was also modified as  $\alpha = 0.5/(|\Delta\mathbf{x}|^2 + 0.1|\Delta\boldsymbol{\theta}|^2)$ .

The simulation used two artificial neural networks with four layers. The first layer of the coordinate transformation gains had 7 linear units and the fourth layer had 21 linear units. The other layers had 30 sigmoid units each. It is highly unlikely that the nervous system utilizes the backpropagation learning. However, since the choice of the learning method of the neural networks is not essential for the evaluation, backpropagation learning was used.

$\lambda_{ff}$  and  $\lambda_{fb}$  in Eq. (11) were set at 0.001. The standard deviation of each component of the normal random number vector  $\mathbf{d}(k)$  for the learning based on the disturbance

noise and the feedback error signal was 0.02 rad. That for the learning based on the change of the hand position error was 0.01 rad. Fig. 4 shows the learning progress by the proposed models. 16384 tracking trials were conducted to estimate the RMS (Root Mean Square) of  $e(k)$ . The dashed line with black boxes shows the percentage of successful trials (%). The solid line with white circles shows the RMS error of hand position (m). After ten million learning trials, the RMS error became lower than 0.015 m. The proposed models are capable of learning a correct feedback controller.

## 6. Conclusions

Novel learning models of coordinate transformation of the visual feedback controller considering time delay were proposed and tested in this paper. They are capable of coordinate transformation learning without using a forward model or the complex signal switching.

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